Bayesian estimation of customer equity from survey data

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April 22, 2013

Abstract

We present a Bayesian framework for estimating the customer equity and the customer lifetime value (CLV) based on the purchasing behaviour deducible from the market surveys. We analyse a consumer survey on mobile phones carried out in Finland in February 2013. The survey data contains consumer given information on the current and previous brand of the phone and the times of the last two purchases. In contrast to personal purchase histories stored in a customer registry of a company, the survey provides information also on the purchase behaviour of the customers of the competitors. The proposed framework systematically takes into account the prior information and the sampling variance of the survey data and by using Bayesian statistics quantifies the uncertainty of the customer equity and CLV estimates by

posterior distributions. The introduced approach is directly applicable in the domains where a customer relationship can be thought to be monogamous.

Keywords: Bayesian estimation, Brand switching, Customer equity, Customer lifetime value, Survey

1 Introduction

In the field of marketing, the monetary value of a future relationship with a customer is a fundamental concept for the rational long-term management of the customer base and the planning of marketing activities. In recent years, a lot of effort has been invested to develop models for estimating the future value of a customer relationship, or *customer lifetime value* (CLV) (Bejou et al., 2006; Gupta and Lehmann, 2005; Blattberg et al., 2008). Thanks to this development, companies are nowadays able to utilize forward looking monetary estimates when assessing the value of the company (Bauer et al., 2003; Pfeifer, 2011), planning marketing actions (Kumar et al., 2008) or optimizing the customer base and allocating their resources (Venkatesan and Kumar, 2004a; Kumar and Petersen, 2005; Venkatesan and Kumar, 2004b). If applied properly, CLV and the future value of the total customer base, or *customer equity* (CE) (Kumar and George, 2007), are able to reveal customers that are most valuable to a company in the long run, and direct the actions to produce optimal return on marketing investments.

For companies with an access to customer level purchase histories, CLV can be estimated for each customer, at the *individual level*, based on the data on the past purchases of an individual customer and stochastic modeling of the general purchasing behaviour of the population (Schmittlein et al., 1987; Fader et al., 2005b,a). For many companies, however, purchase histories of individual customers are not available and CLV models based on such data are not applicable. This can be due to many reasons. Many companies producing consumer goods or services use retailers to sell their products and thus lack the direct interaction with their end customers. In some cases, the time interval between repurchases is long, and gathering meaningful individual level purchase histories takes a lot of time. And even in the cases where the individual level customer data would be in principle available, the lack of maturity of company's data gathering process, the strict privacy policies or the prohibitive cost of collecting purchase data might not allow the utilization of the data for involved statistical modeling.

As all companies need to plan their marketing actions and portfolio to maximize return on investment (ROI), also companies without an access to

individual purchase histories would benefit from understanding the average CLV of different customer segments. As an alternative data source to understand customer behaviour, market surveys can be applied. A company may also want to use a survey to complete the individual level purchase data. The data from the customer registry may suffer from adverse selection due to the cohort heterogeneity (Fader and Hardie, 2010) and thus give a biased view on retention. Furthermore, customer churn (end of the customer relationship) cannot be directly observed from the customer register data if there is no contract between the company and the customer. On the other hand, survey data are drawn as a random sample from the whole population and therefore provide unbiased estimates of retention and churn, even in a non-contractual business setting. In addition, in a survey the respondents can be asked also on their purchase intentions, which are not visible from the purchase histories.

Pareto / negative binomial distribution (Pareto/NBD) model (Schmittlein et al., 1987) and Beta-geometric / negative binomial distribution (BG/NBD) model (Fader et al., 2005a) are commonly used to model customer purchase behaviour when complete personal purchase histories are available. Personal purchase histories from surveys are limited because the survey respondents can be realistically assumed to remember only few latest transactions. Despite the limitations we show that survey data are sufficient for the estimation of customer equity with a good accuracy using BG/NBD as conceptual model for customer purchase behaviour. Furthermore, to take a full advantage of a competitor information contained in survey data, we extend the standard BG/NBD model with brand switching modeling.

For a manager responsible for marketing decisions, the value from the CLV modeling comes as a reduction of the expected opportunity loss from a wrong marketing decision (Hubbard, 2010). In other words, to be useful for a marketing manager, a CLV model needs not only give point estimates of the CLV and customer equity of different customer segments, but also to quantify the uncertainty of the estimates. This is especially important for customer equity estimates based on survey data, as surveys cover only a sample of the whole population, and hence contain a stochastic error component absent in data describing the full purchase histories of individual customers. Below we show how to apply Bayesian analysis (Gelman et al., 2003) to estimate the uncertainty in CLV and CE estimates given by the proposed model.

We demonstrate the proposed model for estimating CLV from survey data by analysing a survey with 536 respondents carried out in Finland in February 2013. The respondents provided information on the brand of their current and previous mobile phone and the times of the last two purchases. For each individual, we model the intensity of purchase process and the personal

probability of retention as latent variables and apply weakly informative priors for these quantities.

The exact model combining brand switching to BG/NBD model is presented in Section 2. The estimation of the model parameters is considered in Section 3 where also a simulation example is presented. The simulation example indicates that the total customer equity can be estimated with a sufficient accuracy from survey samples of realistic size. The analysis of the actual survey data is then presented in Section 4. Conclusions are given in Section 5.

2 Purchase model and survey data

We assume that purchase behaviour of the *population* or a strata of the population can described using an extension of the BG/NBD model (Fader et al., 2005a). The assumptions of the model are

1. The number of transactions made by an individual i follows a Poisson process with the transaction rate λ_i.

2. Transaction rate λ_i follows a Gamma distribution with the probability density function

$$f(\lambda_i \mid \gamma, \delta) = \frac{\delta^\gamma}{\Gamma(\gamma)} \lambda_i^{\delta-1} e^{-\delta\lambda_i}, \quad \lambda_i > 0.$$

3. After any transaction, an individual may change the brand with a probability that depends on the current brand. For the focal company, the probability of retention for individual i is p_i. The probability of acquisition, i.e. change from any competitor to the focal company, is $1 - q_i$ for individual i.

4. Retention probability p_i follows Beta(α_p, β_p) distribution and competitor retention probability q_i follows Beta(α_q, β_q) distribution.

The model combines the idea of brand switching to the BG/NBD model. An individual is either in a state where she has made the last transaction with the focal company, or the individual is in a 'competitor state', where she has made the last transaction with one of the competitors of the focal company. The transaction rate is assumed to be the same in the both states and independent on the transition probabilities. The model is suitable for domains where a customer relationship can be thought to be monogamous.

The products in our scope include various electronic devices and household appliances.

Bayesian analysis requires prior distributions to be defined for all parameters. The prior distributions may reflect knowledge obtained from the previous studies and other information sources or they may be weakly informative in which case the posterior is dominated by the data. For instance, a weakly informative prior may reflect the common knowledge that mobile phones can be used for several years and brand switching is rather common.

The data on the purchase behaviour are obtained for a small random sample of the population. The collected data contains answers to the following questions:

1. What is the brand of your current device?
2. When did you buy your current device?
3. What was the brand of your previous device?
4. When did you buy your previous device?

The questions in the survey should be formulated so that the risk of systematic bias is minimized. The data collected for the individuals $i = 1, 2, \ldots, n$ are the current state $S_i^{(0)}$ (1 for the focal company and 0 for the competitors), the previous state $S_i^{(-1)}$, the time between the last two purchases τ_i and the time from the latest transaction τ_i^*.

As the transactions follow the Poisson process, the time between the purchases τ_i follows exponential distribution with rate λ_i. The time from the latest purchase to the day of the survey τ_i^* is an observation from the same exponential distribution because the Poisson process is memoryless. For the current state it holds

$$P(S_i^{(0)} = 1) = p_i S_i^{(-1)} + (1 - q_i)(1 - S_i^{(-1)}). \tag{1}$$

For the previous state it holds

$$P(S_i^{(-1)} = 1) = p_i S_i^{(-2)} + (1 - q_i)(1 - S_i^{(-2)}), \tag{2}$$

where $S_i^{(-2)}$ is the state before the previous. For the state $S_i^{(-2)}$ there are no observations but the formula

$$P(S_i^{(-2)} = 1) = \frac{1 - q_i}{2 - q_i - p_i} \tag{3}$$

follows from the equilibrium state of the Markov chain characterizing the brand switching.

3 Bayesian estimation of customer equity

The procedure for Bayesian estimation of customer equity with survey data has the following steps

1. Specify the prior distributions for γ, δ, α_p, β_p, α_q and β_q of the extended BG/NBD model.

2. Collect survey data $(S_i^{(0)}, S_i^{(-1)}, \tau_i, \tau_i^*)$, $i = 1, 2, \dots, n$.

3. Use Markov chain Monte Carlo (MCMC) or other simulation techniques to generate observations from the joint posterior distribution of the parameters (λ_i, p_i, q_i), $i = 1, 2, \dots, n$.

4. For each set of parameters generated from the joint posterior distribution, generate purchase histories for n individuals and calculate the individual CLVs and the customer equity as their sum. These values are observations from the posterior distribution of the customer equity of the survey sample.

5. Using the knowledge on the size of the market, scale the customer equity distribution of the sample to present the whole customer population.

The estimation method is demonstrated with simulated data from the extended BG/NBD model. Full purchases histories are generated for a population from where small survey samples are drawn. The variables $(S_i^{(0)}, S_i^{(-1)}, \tau_i, \tau_i^*)$, $i = 1, 2, \dots, n$ are recorded for the sample and the customer equity distribution is estimated according to the procedure above. The estimated customer equity is compared with the customer equity of the population. The procedure is repeated for a number of survey samples to obtain information on the sampling variation.

For the numeric example we generate purchase histories for a population of 100,000 individuals who are divided between the focal company and the competitors according to the market shares. The parameters used are $\gamma = 3$, $\delta = 10$, $\alpha_p = 4$, $\beta_p = 6$, $\alpha_q = 4$, $\beta_q = 6$ and the intensity is defined as the number of transactions per year. The value of a purchase assumed to be 100 euros. The purchase histories are generated for the 40 years forward and 30 years backward from the time of the survey. The true CLVs and the customer equity are calculated using the whole population and the generated purchase histories for the forthcoming 40 years. With the annual discounting rate of 10 % this leads to customer equity of 10.0 million euros for the population of the 100,000 individuals and an average CLV of 100 eurors. For the current customers of the focal company, the average CLV equals 120 euros and for the

current customers of competitors, the average CLV equals 91 euros. These numbers are compared to the estimates from a small survey samples from the same population. For the each individual selected to the sample, only the variables $S_i^{(0)}$, $S_i^{(-1)}$, τ_i and τ_i^* are recorded at the time of the survey, which means the amount of the data from the sample is exiguous compared to the full future purchase histories of 100,000 individuals. To illustrate the effect of the sample size to the accuracy of the estimates, the sample sizes are varied from 100 to 1000.

For parameters γ, δ, α_p, β_p, α_q and β_q we use weakly informative prior distributions (Gelman, 2006) that reflect our knowledge on the purchase intervals for mobile phones and are also used in the real data analysis of Section 4. Parameters γ and δ describe the shape and scale of the Gamma distribution where the values for the intensity λ_i are drawn. We define $\gamma = m_\lambda^2/v_\lambda$ and $\delta = m_\lambda/v_\lambda$ where $m_\lambda \sim \text{Gamma}(2, 1)$ is the mean of the intensity distribution and $v_\lambda \sim \text{Gamma}(2, 1)$ is the variance of the intensity distribution. In other words, the expected mean of the intensity distribution is 2 years and the expected standard deviation of the intensity distribution is 1.4 years but there is a considerable uncertainty on the intensity distribution.

Parameters α_p and β_p describe the Beta distribution from where the individual retention probabilities are drawn and parameters α_q and β_q describe the Beta distribution from where the individual competitor retention probabilities are drawn. We define $\alpha_p = k_p m_p$ and $\beta_p = k_p(1 - m_p)$ where $m_p \sim \text{Unif}(0, 1)$ is the expected average retention probability and $k_p \sim \text{Gamma}(10, 1)$ controls the variation of the retention probabilities in the population. Similarly we define $\alpha_q = k_q m_q$ and $\beta_q = k_q(1 - m_q)$ where $m_q \sim \text{Unif}(0, 1)$ and $k_q \sim \text{Gamma}(10, 1)$. These priors for the expected average retention probabilities are uninformative but the $\text{Gamma}(10, 1)$ for k_p and k_q makes sure that there is a reasonable variation of retention probabilities in the population.

The analysis is carried out using OpenBUGS 3.2.2 (Lunn et al., 2009), R (R Core Team, 2012) and R2OpenBUGS R package (Sturtz et al., 2005). The BUGS model can be written as follows

```
model
{
  for(i in 1:N)
  {
    lambda[i] ~ dgamma(gammal,deltal)
    p[i] ~ dbeta(alphap,betap)
    q[i] ~ dbeta(alphaq,betaq)
    tau[i] ~ dexp(lambda[i])
```

```
    taustar[i] ~ dexp(lambda[i])
    m0[i] <- (1-q[i])/(2-q[i]-p[i])
    S2[i] ~ dbern(m0[i])
    S1prob[i] <- p[i]*S2[i]+(1-q[i])*(1-S2[i])
    S1[i] ~ dbern(S1prob[i])
    S0prob[i] <- p[i]*S1[i]+(1-q[i])*(1-S1[i])
    S0[i] ~ dbern(S0prob[i])
  }
  ml ~ dgamma(2,1)
  vl ~ dgamma(2,1)
  gammal <- ml*ml/(vl+0.00001)
  deltal <- ml/(vl+0.00001)
  mp ~ dunif(0,1)
  mq ~ dunif(0,1)
  kp ~ dgamma(10,1)
  kq ~ dgamma(10,1)
  alphap <- kp*mp
  betap <- kp*(1-mp)
  alphaq <- kq*mq
  betaq <- kq*(1-mq)
}
```

The simulation results are shown in Figure 1 and in Table 1. From Figure 1, it can be seen that the estimated posterior distributions are concentrated around the true value of the customer equity and the systematic bias is small or non-existing. As expected, the variance is smaller for the larger sample sizes. Sample sizes of 800 or more seem to give sufficient accuracy of estimation.

The CLV posterior distributions for the customers of the focal company and the customers of a competitor are presented in Table 1. It can be seen that the posteriors estimated from a sample of size 1000 are very similar to the true CLV distribution of the population.

4 Customer equity for mobile phone brands

The mobile phone data was collected in February 2013 together with the National Consumer Net Shopping Study conducted by market research company Tietoykkönen Oy. The target group was 15–79 years old mobile phone owners in Finland. The data collection method was telephone interviews by using a computer-assisted telephone interviewing (CATI) system. The sample source was targeting service Fonecta Finder B2C, which contains all

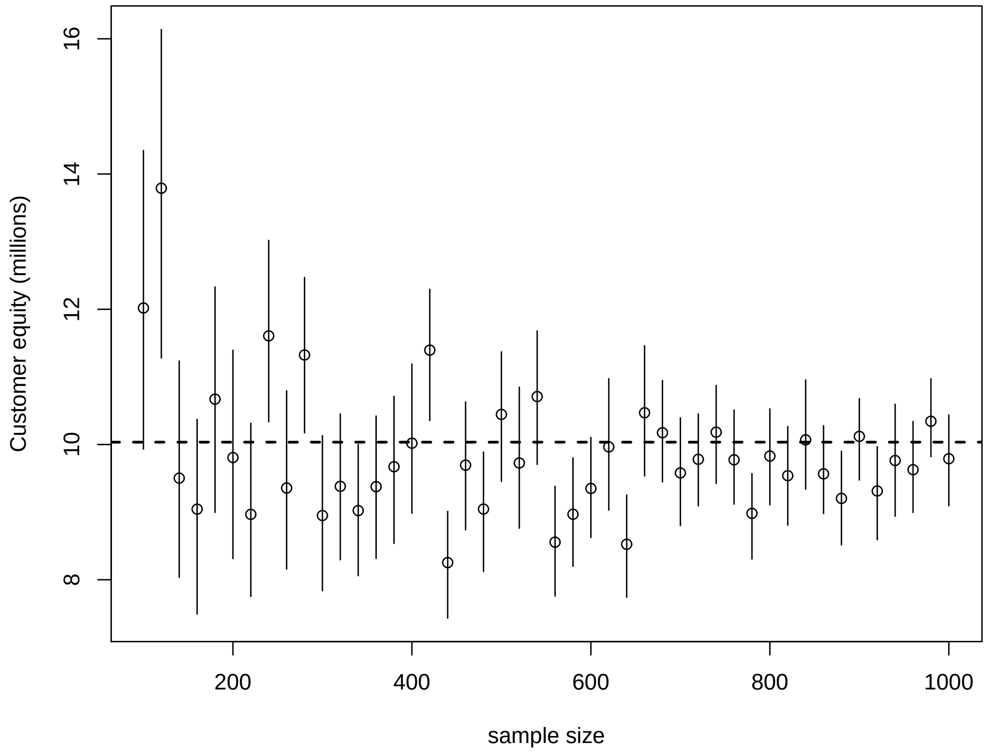

Figure 1: The accuracy of the Bayesian customer equity estimation as a function of the sample size of the survey. The circles show the mean of the estimated customer equity posterior distribution and the vertical lines show the posterior range from the 1st decile to the 9th decile. The horizontal line shows the true customer equity of the population.

Table 1: Estimated CLV distributions for the customers of the focal company and the customers of a competitor in the simulation example.

Last purchase with the focal company

N	Min	1st Qu.	Median	Mean	3rd Qu.	Max
Population	0	33.92	93.93	120.6	173.9	1122
1000	0	35.70	93.65	118.1	169.8	1015
500	0	40.94	101.1	129.2	185.1	1050
100	0	56.66	125.4	150.2	210.7	865

Last purchase with a competitor

N	Min	1st Qu.	Median	Mean	3rd Qu.	Max
Population	0	11.75	62.28	90.57	135.2	1184
1000	0	12.64	62.81	88.11	131.8	966
500	0	12.06	64.38	92.77	139.4	907.7
100	0	16.67	74.64	101.2	152.5	857.6

publicly available phone numbers in Finland. Random sampling was made by setting quotas in respondents gender, age and region in Finlands major region level excluding Åland autonomic region. The sample size was 536 completed interviews. Table 2 shows the structure of the data.

Compared to Finnish official statistics (Statistics Finland, 2012) the amount of women and youngest age group (15–24 years) in the data is slightly too small and men and oldest age group (65–79 years) too large. All 536 respondents had a mobile phone. The respondents answered the following questions (originally in Finnish):

1. What is the brand of your mobile phone?
2. When did you purchase your mobile phone? (year and month; if the month was not recalled the season was asked)
3. What was the brand of your previous mobile phone?
4. When did you purchase your previous mobile phone? (year and month; if the month was not recalled the season was asked)
5. Which brand would be the most interesting for you if you were to buy a mobile phone now?
6. Is your mobile phone a smart phone, a feature phone with an internet connection or a phone without an internet connection?

Table 2: Respondents of the mobile phone survey by gender, age and major region

		Sample size	Proportion in sample	Proportion in population (age 15–79 y)
Gender	Man	285	53%	50%
	Woman	251	47%	50%
Age	15–24 years	63	12%	16%
	25–34 years	88	16%	16%
	35–44 years	73	14%	16%
	45–54 years	89	17%	18%
	55–64 years	103	19%	18%
	65–79 years	120	22%	17%
Region	Helsinki-Uusimaa	143	27%	29%
	Southern	91	17%	22%
	Western	155	29%	27%
	Northern and Eastern	147	27%	23%
Total		536	100%	100%

Table 3 presents the distribution of the brand by gender and age. It can be seen that there is a clear association between the age group and the current brand. Nokia still has a dominant position in Finland but its share of the installed base varies from the 93% of the age group 65–79 years to the 46 % of the age group 15–24 years. In contrast, Apple and Samsung are relatively strong in the younger age groups. This suggest that the analysis should be stratified by the age group.

The purchase times are interval censored: the respondents are asked only for the purchase month, not for the day and many respondents could not recall the time of the purchase. Out of 536 respondents, 310 were able to tell the purchase month and year, additional 115 were able to tell the season and year, 74 mentioned only the year and 37 were not able to tell even the year. 19 respondents told that they did not have a mobile phone earlier or they cannot remember the brand. Out of 517 respondents who mentioned the previous brand, 117 were able to tell the purchase month and year, additional 91 were able to tell the season and the year, 146 mentioned only the year and 163 were not able to tell even the year. The minimum and maximum of the purchase intervals τ_i and τ_i^* are calculated for all individuals using the available data.

Table 3: Brand of current mobile phone by gender and age

		Current brand			
		Nokia	Apple	Samsung	Others, n/a
Gender	Man	70%	6%	18%	6%
	Woman	76%	9%	11%	4%
Age	15–24 years	46%	14%	27%	13%
	25–34 years	52%	16%	26%	6%
	35–44 years	71%	3%	22%	4%
	45–54 years	78%	6%	13%	3%
	55–64 years	81%	8%	8%	4%
	65–79 years	93%	2%	3%	3%
Total		73%	7%	15%	5%

The maximum lifetime of 200 months is assumed when the purchase year is missing. The amount of the missing data is large for the previous purchase but this does not jeopardize the analysis because the estimation could be carried out using solely τ_i^* which requires only the purchase time of the current phone. It is assumed that on the condition of τ_i^*, the missingness of τ_i does not depend on the value of τ_i. Figure 2 shows the purchase time of the current phone and the time between the last purchases.

Information on the sell-in prices is needed for the estimation and is collected from the quarterly reports of Nokia, Apple and Samsung. For the 4th quarter of 2012, Nokia reported average sales price (ASP) 186 euros for smart phones and 31 euros for (other) mobile phones . For the same period, the ASP for Apple was 641 US dollars (473 euros) and the ASP for Samsung 178 euros. As the actual ASPs for Finland have not been published, these global numbers are used in the CLV estimation.

The BUGS code used in the analysis is similar to the code presented in Section 3 with two exceptions: the hyperparameters are stratified by the age group and the interval censoring is accounted by specifying

```
tau[i] ~ dexp(lambda[i])C(tau_min[i],tau_max[i])
taustar[i] ~ dexp(lambda[i])C(taustar_min[i],taustar_max[i])
```

. It is assumed that mobile phones are puchased up to the age 80. This implies that the younger age groups are a priori more valuable for the companies. The exact valuation depends also on the age group specific purchase frequencies and the discount rate, which is assumed to be 10 % per annum. The estimation is carried out using OpenBUGS 3.2.2 (Lunn et al., 2009),

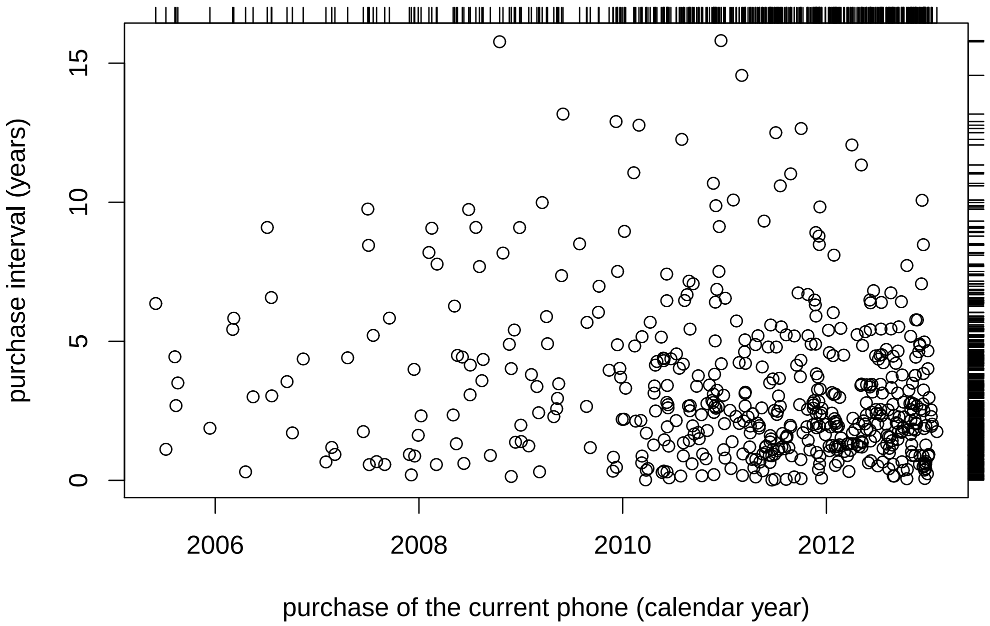

Figure 2: The purchase time of the current phone and the time between the last purchases. For the illustration, the interval censored times have been imputed with draws from the posterior distribution. Twenty individuals who have bought their current phone prior to the year 2005 are not displayed.

R (R Core Team, 2012) and R2OpenBUGS R package (Sturtz et al., 2005). The convergence of the MCMC chains is monitored separately for each parameter using the interval criterion proposed by Brooks and Gelman (1998).

Figure 3 presents the estimated average CLVs for the six age groups. The average CLVs are estimated both for a current customer of the focal company and for a current customer of a competitor. As expected the younger age groups have higher average CLV than the older age groups for all companies. For Apple, the differences in CLV between the current customers (those who currently own an Apple phone) and the non-customers (those who currently do not own an Apple phone) seem to be large whereas for Nokia the differences are small. This can be explained so that Apple has high retention probabilities and lower acquisition probabilities while for Nokia the retention and the acquisition probabilities are close to each other. The results also show that there is lot of uncertainty related to the average CLV of the current Apple owners because only 40 respondents currently own an Apple phone.

The survey question "Which brand would be the most interesting for you if you were to buy a mobile phone now?" allows an alternative analysis where the retention and acquisition probabilities are estimated on the basis of the current brand and the most interesting brand for the next phone. 450 respondents provided an answer to this question. Figure 4 presents the estimated average CLVs in the alternative analysis. There are clear differences compared to the results in Figure 3. The average CLVs for the current Apple owners are very high and also for Nokia and Samsung the current customers are clearly more valuable than non-customers. In other words, consumers are interested in buying the brand they currently own. The forward-looking analysis shows smaller CLVs for Nokia and larger CLVs for Apple and Samsung than the historical analysis. The actual purchase decision may naturally differ from the reported interest.

In Figure 5, both the historical and the forward-looking estimate of the customer equity are reported for the Finnish population of age 15–79. Thanks to the high number of current customers, the position of Nokia remains strong in the age groups 35–79 years. In the age groups 15–34 years where the customer equities are the highest, Apple and Samsung have relatively strong position.

5 Discussion

We have presented a Bayesian approach to estimate the average CLV and the customer equity on the basis of survey data. The presented approach

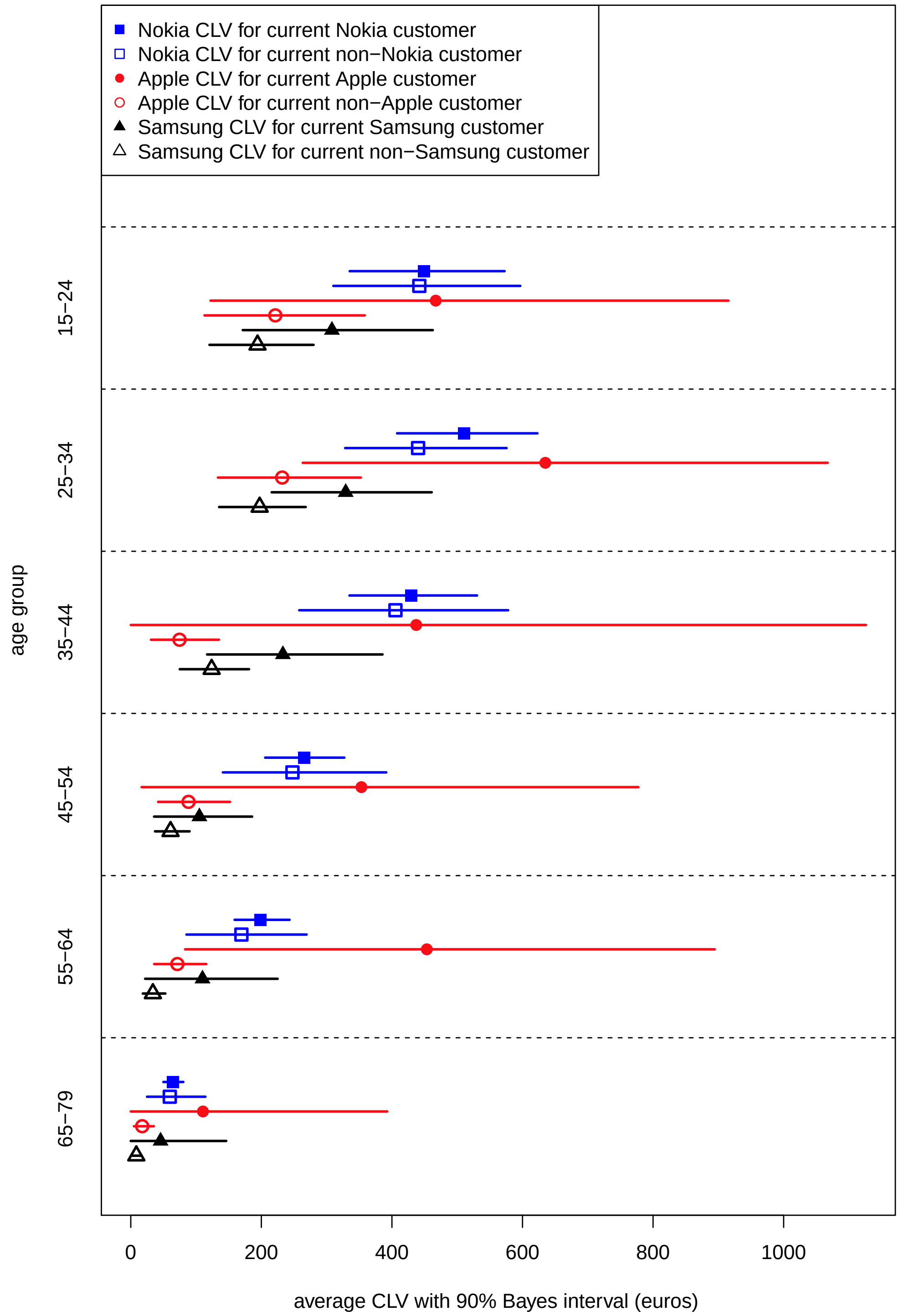

Figure 3: The average CLVs estimated with the historical retention probabilities.

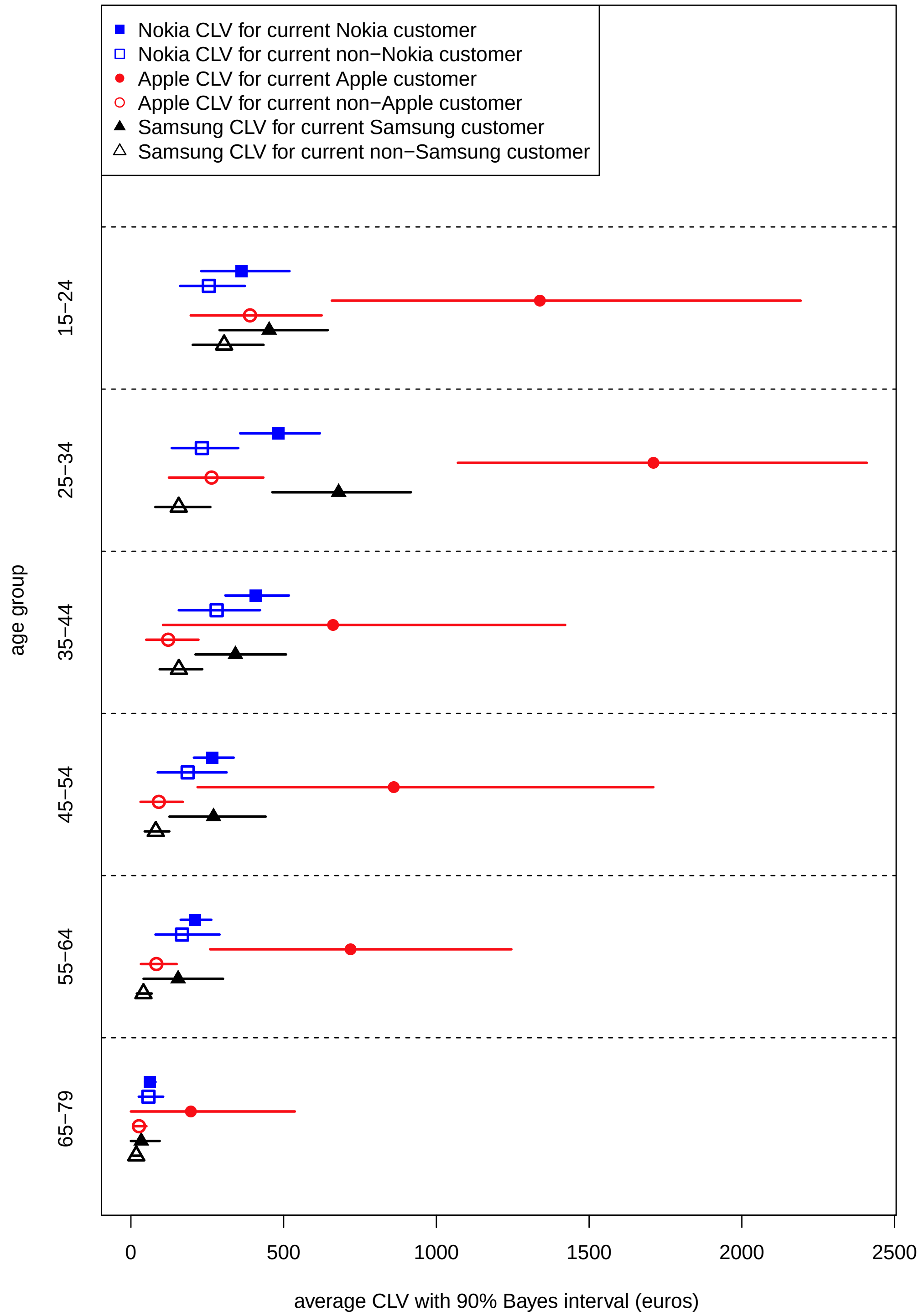

Figure 4: The average CLVs estimated with the forward-looking retention probabilities.

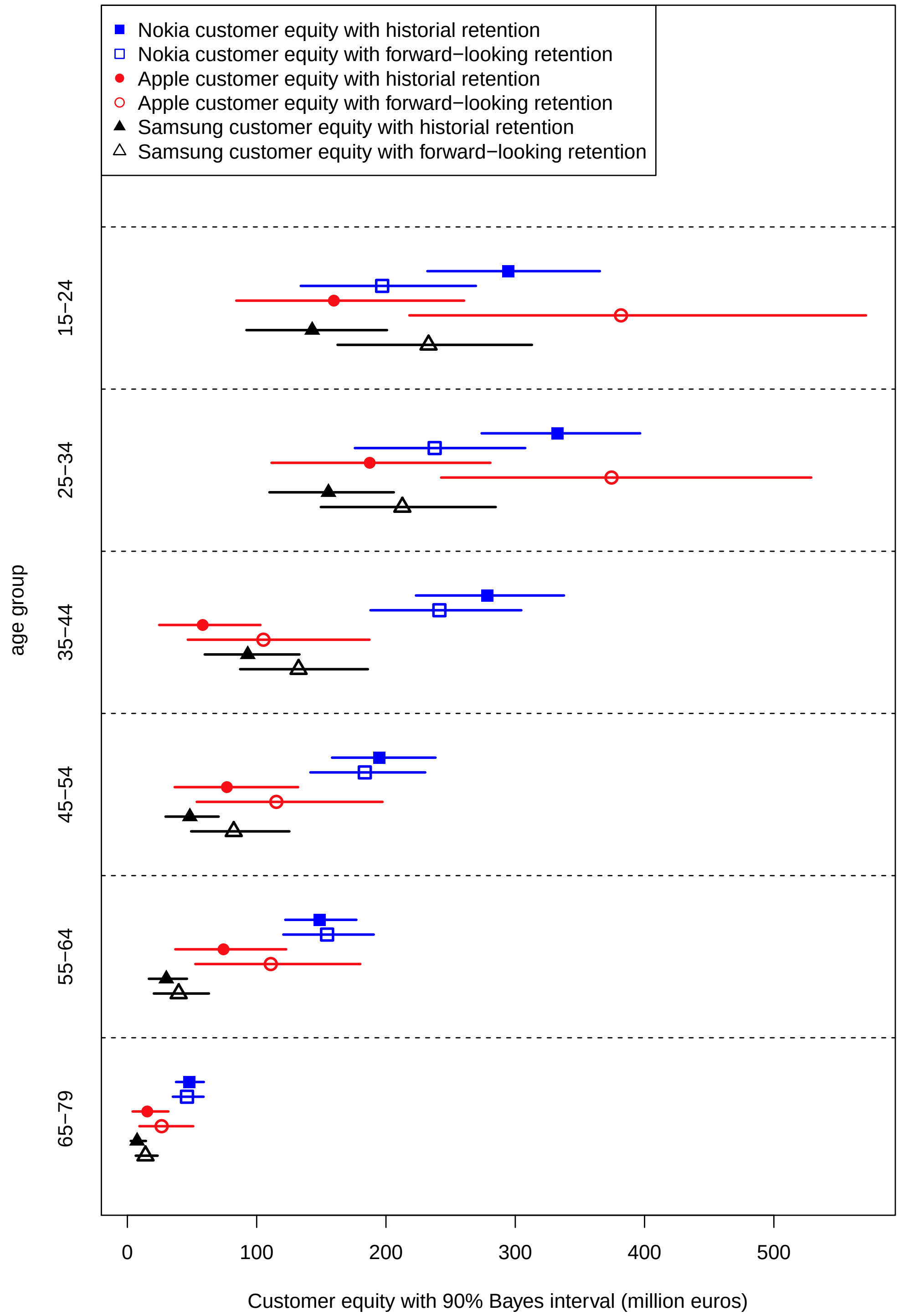

Figure 5: The customer equities in the historical and forward-looking analysis.

is motivated by the need for the CLV based decision making in the absence of personal purchase histories. Most companies manufacturing consumer goods do not directly sell the goods to their end customer, and hence are unable to directly collect transactional purchase data at the level of individual customer. For these companies, surveys offer a natural option to obtain information about the purchasing behaviour different customer segments.

Although the amount of the data is significantly smaller in surveys compared to customer registries, the survey based approach for the estimation of customer equity has important benefits. A survey gives insight also on the purchase behaviour of the customers of the competitors. A properly collected sample represents the population and thus avoids the problems of cohort heterogeneity. Carrying out a survey is usually an easier option than organizing systematic collection of purchase histories and does not require investments to transactional data collection and storage.

As always in survey sampling, the validity of the results depends on the representativeness of the sample and major differences in the survey response rates between the customer segments may bias the customer equity estimates. In many cases, the bias can be removed or reduced by stratified sampling or post-stratification which lead to unequal weighting of the individuals in the sample.

The CLV estimation is not a straightforward task because the CLV is not directly observed but depends on latent purchase intensities and retention probabilities. Bayesian approach with weakly informative priors is a natural choice for the estimation of latent variables. After the posterior distributions of the latent variables have been estimated, the posterior distribution of CLV can be estimated with a forward simulation where the intensity and probability parameters are drawn from their joint posterior distribution.

The CLV should be understood as a projection of the current state to the future, not as an attempt to forecast the market development in the future. Disruptive innovations, such as the introduction of Apple iPhone in 2007, are difficult to predict but may have a major impact to the future retention probabilities.

We believe that the presented Bayesian approach and its application will help in advocating the usefulness of the CLV modeling even in the absence of personal purchase histories. Furthermore, we believe that the attention to the uncertainty of the CLV will make the risks more explicit for the decision makers.

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